Rossmoyne Senior High School

Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 3

SOLUTIONS

Section Two: Calculator-assumed

WA student number: In figures | | | | | | | | |

In words _____

_____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	12	12	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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Section Two: Calculator-assumed **65% (98 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

See next page

Question 8 (7 marks)

Mark has started an emu farm down in the Southwest of Western Australia. Each year he does a stocktake of the number of birds he has on the property. The population of birds he had for the first three years is shown in the table below. The number of birds is rounded to the nearest whole number.

Year	Start Population	End population
1	85	72
2	72	61
3	61	52

(a) Show that the recurrence relation for the above data is

$P_{n+1} = 0.85\ P_n \qquad P_0 = 85$. (2 marks)

Solution
(a) Initial population is 85✓ $r = \frac{72}{85} = \frac{61}{72} = 0.847 \therefore 0.85\ (2\text{dp})$ ✓
Mentions initial population Justifies 0.85

(b) Describe the change in the population of birds for each year. (2 marks)

Solution
15% decrease ✓
Must mention decrease

(c) Determine the number of birds he will have at the end of five years. (1 mark)

Solution
37.71 or 37.57 using table $\therefore$ 38 ✓
Must round to the nearest whole number as per question above

(d) After five years, Mark decides to contact his friend who has a similar farm and is willing to give Mark ten birds per year. By finding the steady–state solution, determine the effect this will have on Mark's bird population in the long term. (2 marks)

Solution
$x = 0.85x + 10$ $\therefore$ x = 66.7 ✓ $\therefore$ he will maintain 66 or 67 birds ✓
Must round to the nearest whole number if taken in (c) do not take again

See next page

Question 9 **(7 marks)**

(a) The monthly units of electricity u consumed by each apartment in a building was strongly associated with the average monthly maximum temperature, T °C. The least-squares line for the variables was $\hat{u} = 88.5 + 2.7T$.

(i) Predict the units of electricity consumed by an apartment in a month when the average monthly maximum temperature was 33°C. (1 mark)

Solution
$\hat{u} = 88.5 + 2.7(33) = 177.6$ units.
Specific behaviours
✓ correct prediction

(ii) In a month when the average monthly maximum temperature was 25°C, an apartment consumed 153.4 units of electricity. Calculate the residual for this data point. (2 marks)

Solution
$\hat{u} = 88.5 + 2.7(25) = 156$ Residual: $u - \hat{u} = 153.4 - 156 = -2.6$ units.
Specific behaviours
✓ indicates correct $\hat{u}$ ✓ correct residual

(b) In a government study, the correlation coefficient for the association between age and superannuation balance for employed adults was found to be 0.755. What percentage of the variation in superannuation balance for employed adults is unexplained by their variation in age? (2 marks)

Solution
$r^2 = 0.755^2 = 0.57$ Since 57% of the variation is explained, then 43% is unexplained.
Specific behaviours
✓ calculates coefficient of determination ✓ correct percentage

(c) After measuring the age and hearing acuity of a group of pensioners, a researcher observed a negative linear association between the variables and found that 61% of the variation in hearing acuity can be explained by the variation in age. Determine the correlation coefficient for the association. (2 marks)

Solution
$r^2 = 0.61 \rightarrow r = \pm\sqrt{0.61} = \pm 0.781$ Since association is negative, then $r = -0.781$.
Specific behaviours
✓ indicates square root of coefficient of determination ✓ correct correlation coefficient

See next page

Question 10 **(8 marks)**

A random sample of adults who were not working and not seeking work were recently asked for the main reason that they were not looking for work. The responses, categorised by the sex of the adult and their main reason, are summarised in the table below.

Reason	Male	Female
Education	143	116
Family considerations	33	117
Other	84	37

(a) How many adults gave a response? (1 mark)

Solution

$143 + 33 + 84 + 116 + 117 + 37 = 530$ adults.

Specific behaviours

✓ correct total

(b) What percentage of the females gave education as their main reason? (2 marks)

Solution

$116 + 117 + 37 = 270$

$116 \div 270 = 43\%$

Specific behaviours

✓ indicates use of correct figures
✓ any percentage that rounds to 43%

(c) Construct a table showing column percentages for the above data, rounding entries to the nearest whole number. (3 marks)

Solution

M_{TOT}: $530 - 270 = 260$, $143 \div 260 = 55\%$, etc

Reason	Male (%)	Female (%)
Education	55	43
Family considerations	13	43
Other	32	14

Specific behaviours

✓ neat table with row and column headings
✓ both columns add to 100
✓ both columns correct

(d) Discuss whether the data from the survey suggests the presence of an association between the variables sex and reason. (2 marks)

Solution

The data does suggest an association exists between the variables as the pairs of percentages in each row (i.e., for each reason) are quite different for males and females.

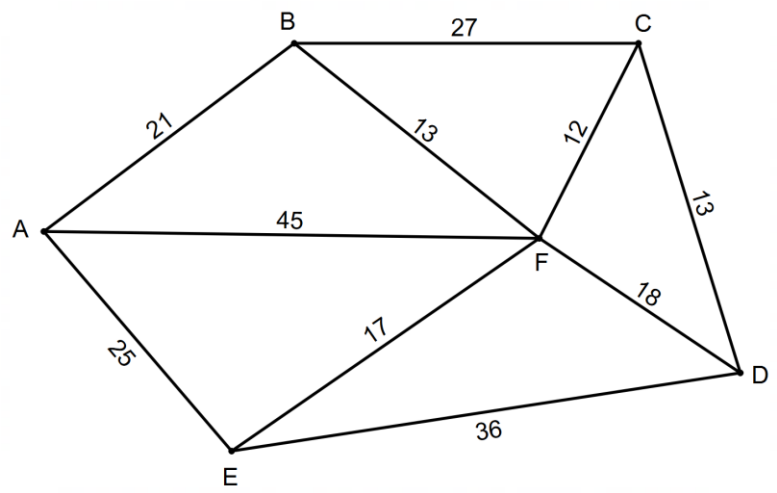
Specific behaviours

✓ states association exists/ or through answer
✓ explains using difference in percentages across reason categories

See next page

Question 11 **(7 marks)**

The graph below represents a cycle pathway around a city. Each weight represents kilometres.



(a) Determine the shortest time and path to travel from A to D. (2 marks)

Solution
A B F D ✓ 52km ✓
Units question

(b) Suggest a possible Hamiltonian Cycle. (2 marks)

Solution
Any possible cycle ✓ Hamiltonian A B C D F E A ✓
Other solutions may exist

Any paths that are longer than 25 kilometres are going to be closed due to the high maintenance costs.

(c) (i) State which roads will be closed. (1 mark)

Solution
AF, BC, ED ✓

(ii) When the paths are closed, all the remaining paths need be cleaned, by a mechanical sweeper, in one continuous cycle without going over the same path. State a trail starting from E. (2 marks)

Solution
All paths covered, cycle ✓ Start and finish (E) E A B F C D F E ✓
Other solutions may exist

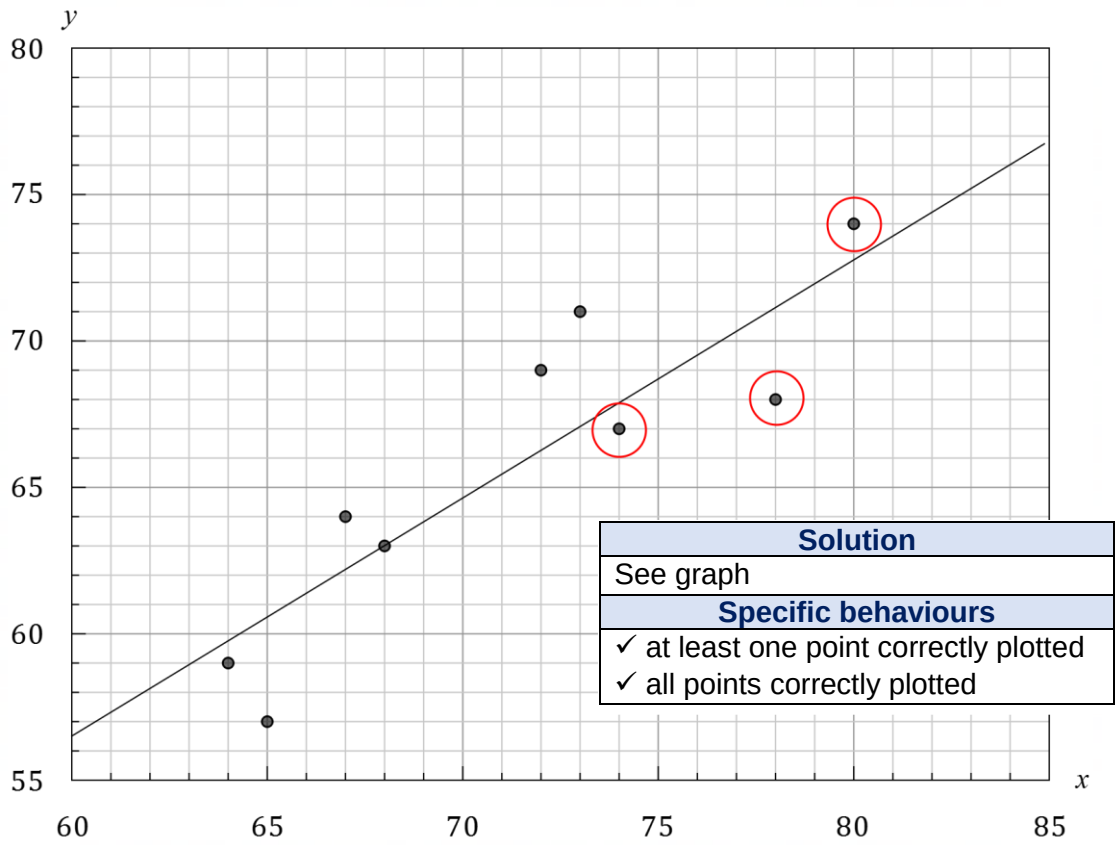
See next page

Question 12 **(13 marks)**

The table below shows the life expectancy, in years, of females and males in nine countries in Oceania.

Country	Female (x)	Male (y)
Federated States of Micronesia	72	69
Kiribati	64	59
Marshall Islands	73	71
Nauru	65	57
New Caledonia	80	74
Palau	78	68
Papua New Guinea	68	63
Solomon Islands	74	67
Tuvalu	67	64

(a) On the scatterplot below, plot the three missing data points from the table. (2 marks)



Solution
See graph
Specific behaviours
✓ at least one point correctly plotted
✓ all points correctly plotted

(b) Determine the coefficient of determination between the variables and interpret its value in the context of the question. (2 marks)

Solution
$r^2 = 0.7947 \approx 0.79$ 79% of the variation in the male life expectancy can be explained by the variation in the female life expectancy.
Specific behaviours
✓ correct coefficient (to at least 2 dp)
✓ correct interpretation

See next page

(c) State the correlation coefficient between the variables and use its value to comment on the strength of the linear association between female and male life expectancy for these countries. (2 marks)

Solution
$r = 0.891$ The linear association between the variables is strong.
Specific behaviours
✓ correct value of r ✓ states association is strong

(d) Determine the equation of the least-squares line to model the relationship between the variables and draw this line on the scatterplot. (3 marks)

Solution
$y = 0.877x + 3.294$
Specific behaviours
✓ correct equation (coefficients to at least 2 dp) ✓ any line of best fit drawn with a ruler ✓ ruled lined through $(60, 56)$ & $(85, 78)$

(e) The life expectancy of a female from Fiji is 70. Predict, to the nearest year, the life expectancy of a male from the same country and comment on any factors that affect the validity of your prediction. (2 marks)

Solution
$\hat{y}(70) = 64.7 \approx 65$ With strong correlation and the age lying within the range of data, it is reasonable to assume the prediction is valid.
Specific behaviours
✓ prediction ✓ notes valid, with at least one reason

(f) The life expectancy of a female Australian is 86. Predict, to the nearest year, the life expectancy of a male Australian and comment on any factors that affect the validity of your prediction. (2 marks)

Solution
$\hat{y}(86) = 78.7 \approx 79$ Despite the strong correlation, this prediction involves extrapolation and should be treated with caution.
Specific behaviours
✓ prediction ✓ notes dangers of extrapolation

See next page

Question 13 **(8 marks)**

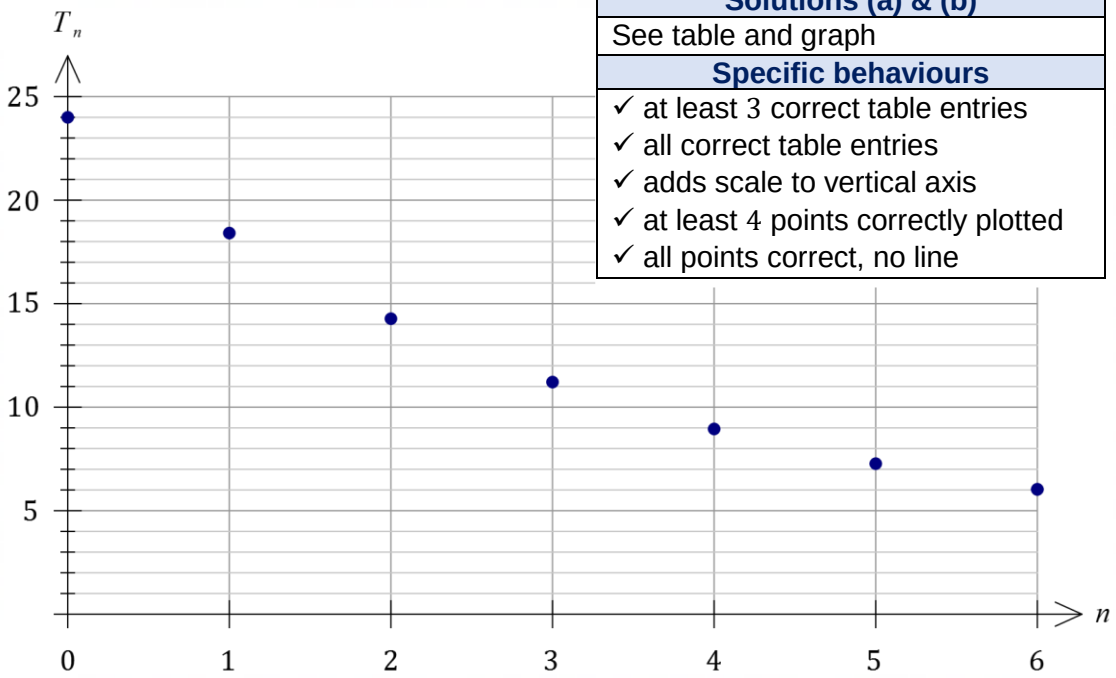
The cooling system for a mobile cool room has just been turned on. The temperature T_n °C inside the cool room, n hours later, is modelled by the linear recurrence relation

$$T_n = 0.74T_n + 0.65, \qquad T_0 = 24.$$

(a) Complete the table of temperatures below. (2 marks)

n	0	1	2	3	4	5	6
T_n (°C)	24.0	18.4	**14.3**	**11.2**	**8.9**	**7.3**	**6.0**

(b) Add a scale to the vertical axis below and then plot the temperature inside the cool room every hour. (3 marks)



Solutions (a) & (b)
See table and graph
Specific behaviours
✓ at least 3 correct table entries
✓ all correct table entries
✓ adds scale to vertical axis
✓ at least 4 points correctly plotted
✓ all points correct, no line

(c) After how many hours does the model predict that the temperature inside the cool room will first reach within 0.1° of its steady state? Justify your answer. (3 marks)

Solution
Using sequence, steady state temperature is 2.5° C. Hence temperature must fall to 2.6° C or below. From sequence, $T_{17} = 2.629$ and $T_{18} = 2.595$ and so cool room will first reach required temperature after 18 hours.
Specific behaviours
✓ indicates steady state temperature
✓ states correct number of hours
✓ justifies time using terms of sequence

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Question 14 **(6 marks)**

The mass of a small puppy was measured as 375 g when it was one week old. A week later its mass had increased by 45 g.

(a) Assuming that the weekly mass of the puppy can be modelled by an arithmetic sequence, predict the mass of the puppy when it is 9 weeks old. (2 marks)

Solution
$T_n = 375 + 45(n-1)$ $T_8 = 375 + 45 \times (9-1)$ $= 735$ g
Specific behaviours
✓ indicates appropriate method ✓ correct mass

(b) Assuming that the weekly mass of the puppy can be modelled by a geometric sequence, predict the mass of the puppy when it is 9 weeks old. (3 marks)

Solution
$r = \dfrac{375+45}{375} = 1.12$ $T_n = 375(1.12)^{n-1}$ $T_8 = 375(1.12)^{9-1}$ $= 928$ g
Specific behaviours
✓ indicates common ratio ✓ indicates appropriate method ✓ correct mass

(c) Comment on the usefulness of these models as the puppy gets older. (1 mark)

Solution
Not very useful, since both models have the mass of the puppy increasing for ever, yet all dogs reach their adult weight after a year or two.
Specific behaviours
✓ sensible comment that notes models eventually not useful

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Question 15 **(10 marks)**

An industrial chemist varied the amount of accelerant (a grams) used when making an epoxy resin and recorded the time taken (t seconds) for the resin to set. Some of the results are shown below.

a	4.5	5.5	6.5	7.0	8.0	9.0	10.0	11.5	13.0	14.0
t	24.1	19.2	19.3	21.8	15.7	19.2	14.8	17.7	15.0	12.3

The chemist suspected that a linear association might exist between the variables and calculated the correlation coefficient $r_{at} = -0.81$.

(a) After seeing this value of the correlation coefficient, the chemist said to their assistant "it looks like there is a strong and negative linear association between the variables". Explain why the chemist made this statement. (2 marks)

Solution
Since r is close to -1, the strength of the linear association is strong. Since $r < 0$, the direction of the linear association is negative. Or: as a increases t decreases
Specific behaviours
✓ explains strong ✓ explains negative

The chemist also noted that the least-squares line for the data was $\hat{t} = 25.78 - 0.896a$ and used it to calculate nine residuals for the linear model as shown below, rounded to one decimal place.

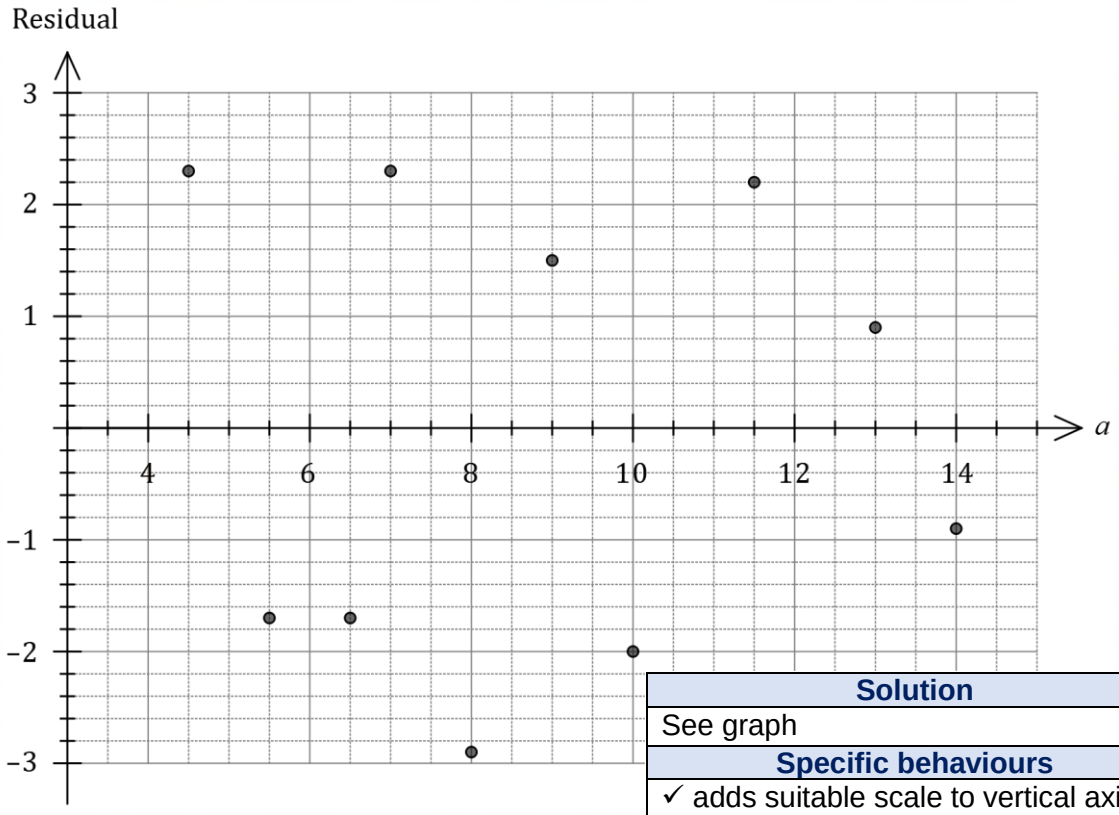
a	4.5	5.5	6.5	7.0	8.0	9.0	10.0	11.5	13.0	14.0
Residual	2.3	-1.7	-1.7	**2.3**	-2.9	1.5	-2.0	2.2	0.9	-0.9

(b) Show how the residual of -2.9 was calculated and determine the residual associated with 7.0 grams of accelerant. (3 marks)

Solution
$\hat{t}(8.0) = 25.78 - 0.896(8.0) = 18.6, \quad 15.7 - 18.6 = -2.9$ $\hat{t}(7.0) = 25.78 - 0.896(7.0) = 19.5, \quad 21.8 - 19.5 = 2.3$
Specific behaviours
✓ calculates $\hat{t}(8.0)$ and subtracts from 15.7 ✓ shows calculation for $\hat{t}(7.0)$ ✓ calculates missing residual

See next page

(c) Construct a residual plot for the data on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ adds suitable scale to vertical axis ✓ accurately plots at least 7 points ✓ accurately plots all points

(d) Does the residual plot support the chemist's suspicions that a linear model fits the data? Explain your answer. (2 marks)

Solution
The residual plot supports the linear model as no pattern is evident in the residuals.
Specific behaviours
✓ states plot supports linear model ✓ states no pattern evident in residuals

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Question 16 **(7 marks)**

A business bought a mainframe computer valued at $\$95\,000$. The value of the computer depreciated by 35% each year.

(a) By how much did the value of the computer depreciate during the first year and what was its value one year after it was bought? (2 marks)

Solution
$0.35 \times \$95\,000 = \$33\,250$ $V_1 = \$95\,000 - \$33\,250 = \$61\,750$
Specific behaviours
✓ depreciation amount ✓ value after one year

(b) Deduce a recursive rule for V_n, the value of the computer after n years. (2 marks)

Solution
$V_{n+1} = 0.65V_n, \quad V_0 = 95\,000$
Specific behaviours
✓ indicates correct multiplier ✓ correct rule with initial term

(c) Calculate the value of the computer after 4 years. (1 mark)

Solution
$V_4 = \$16\,958.09$
Specific behaviours
✓ correct value (accept reasonable rounding)

(d) During which year does the value of the computer **first depreciate** by less than $\$1000$? Justify your answer. (2 marks)

Solution
$V_8 = 3027.13, \quad V_9 = 1967.63, \quad V_{10} = 1278.96$ By observing terms of the sequence, the annual depreciation will first be less than $\$1000$ during the 10^{th} year.
Specific behaviours
✓ indicates appropriate reasoning ✓ correct year, with reasoning

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Question 17 **(8 marks)**

The annual number of mobile phone subscriptions and new cars sold in Western Australia, as collated by a researcher, are shown in the table below.

Year	2011	2012	2013	2014	2015	2016	2017
Subscriptions (s, in millions)	2.93	3.01	3.06	3.14	3.19	3.25	3.31
New cars (c, in thousands)	109	111	112	114	116	117	119

The researcher wanted to identify whether new car sales in Western Australia could be predicted from mobile phone subscriptions.

(a) Quantify the strength of the linear association between the variables s and c. (1 mark)

Solution
$r = 0.997$
Specific behaviours
✓ calculates r

(b) Determine the equation of the least-squares line that can be used to predict c from s. (2 marks)

Solution
$\hat{c} = 26.24s + 31.9$
Specific behaviours
✓ correct coefficients
✓ uses given variables

(c) Use the least-squares line to predict the number of new car sales in another Australian state that had 3.09 million mobile phone subscriptions, and comment, with reasons, on the validity of your prediction. (3 marks)

Solution
$\hat{c} = 26.24(3.09) + 31.9 = 113$ Sales predicted to be 113 thousand cars. This prediction is not valid as the line is derived from sales of cars and smartphones in WA. In another state, it is unlikely that the variables will have the same association as in WA.
Specific behaviours
✓ correct prediction, noting units ✓ states, with reasoning, that prediction not valid ✓ supplies reason that prediction not valid

(d) Describe a possible non-causal explanation for the observed association between mobile phone subscriptions and new cars sold. (2 marks)

Solution
The association is likely due to a common response to a third variable - the population of the state. As the population increases, so there will be more people to buy cars and mobile phone subscriptions.
Specific behaviours
✓ identifies a confounding variable such as population (*do not accept time*) ✓ explains common response to confounding variable

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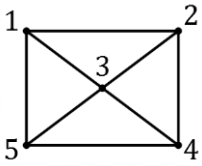
Question 18 **(9 marks)**

The adjacency matrix for the connected planar graph P is $\begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$.

(a) Determine, with justification, the number of faces that P has. (3 marks)

Solution
 P has 5 faces.
Specific behaviours
✓ draws P, correct v and e ✓ draws P in the plane ✓ states number of faces

Alternative Solution
Vertices $v = 5$ (number of matrix rows) and edges $e = 8$ (sum of elements above matrix diagonal). P is a connected planar graph, so using Euler's formula then $5 + f - 8 = 2$ and so $f = 2 + 8 - 5 = 5$. P has 5 faces.
Specific behaviours
✓ states number of vertices and edges ✓ uses Euler's formula ✓ states number of faces

(b) Use elements from the adjacency matrix to explain why P is a simple graph. (3 marks)

Solution
Elements on the leading diagonal are all 0 and so there are no loops. All other elements in the matrix are 0 or 1 and so there are no multiple edges.
Specific behaviours
✓ states no loops and no multiple edges ✓ uses diagonal elements to justify no loops ✓ uses other elements to justify no multiple edges

See next page

(c) Ore's theorem states that a simple graph with n vertices is Hamiltonian if, for every pair of distinct vertices V_a and V_b which are not adjacent, the sum of the degrees of V_a and V_b is greater than or equal to n. Use Ore's theorem to show that P is Hamiltonian. (3 marks)

Solution
P has two pairs of vertices that are not adjacent: V_1 and V_4 with degrees 3 and 3 respectively. V_2 and V_5 with degrees 3 and 3 respectively. For each pair, $3 + 3 = 6$ and 6 is greater than $n = 5$, the number of vertices, and so P is Hamiltonian. (*NB Using adjacency matrix, non-adjacent pairs identified by* 0 *elements not on leading diagonal, and degree is sum of elements in row.*)
Specific behaviours
✓ identifies the two pairs of vertices that are not adjacent ✓ states degrees of both pairs of vertices ✓ shows sum of both pairs is equal to n and states conclusion

See next page

Question 19 **(8 marks)**

John is a rower who rows four times during a month. At the end of each month he increases the distance rowed by a ratio and then adds a constant amount to establish his goal for the following month. In the first month, he rowed a distance of 5 kilometres each time he rowed. In the second month, the distance increased to 5600 metres for each row and for the third month he was rowing
6260 metres for each row.

(a) Write the first order recurrence relation for the above data. (3 marks)

Solution
$T_2 = rT_1 + d$ $\therefore \quad 5600 = 500r + d$ $T_3 = rT_2 + d$ $\therefore \quad 6260 = 5600 + d$ $\therefore \quad T_{n+1} = 1.1T_n + 100 \quad T_1 = 5000$
First equation ✓ Second equation ✓ Correct recurrence relation (award follow through marks) ✓

(b) Determine how many kilometres he would row during the fifth month. (2 marks)

Solution
7.784km
Correct amount ✓ Units ✓

(c) Once John reaches ten kilometres he will stop increasing the distance.
How many months of training does John complete before first reaching this level? (1 mark)

Solution
8 months ✓ (10.692km)

(d) Determine the total distance John has rowed after five months of training. (2 marks)

Solution
Sum of T_1 to T_5 = 31.631km ✓ Four times a month = 4×31.631 $\therefore$ 126.524km
Recognises sum of sequence ✓ Correct ✓

End of questions

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Supplementary page

Question number: _____